Heavy-light form factors: The Isgur-Wise function in point-form relativistic quantum mechanics

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Though the point-form of relativistic quantum dynamics is the least explored of the three common forms of relativistic dynamics, it has several properties that makes it well suited for applications to hadronic physics. Its main characteristics are that interaction terms (if present) enter all four components of the 4-momentum operator, whereas the generators of Lorentz transformations stay free of interactions. As a particular example we are going to present the calculation of electroweak form factors of heavy-light mesons within a constituent-quark model. Since the dependence of matrix elements on the heavy-quark mass is rather obvious in point-form relativistic quantum mechanics, it is comparably easy to study the heavy-quark symmetry and its breaking due to finite masses of the heavy quarks.

Starting point of our investigations are the physical processes from which such electroweak form factors are extracted, i.e. elastic electron-meson scattering and the weak decay of heavy-light mesons. We use a coupled-channel framework in which the dynamics of the intemediate gauge bosons – either photon or W-boson – is fully taken into account. Poincaré invariance is ensured by emplyoing the Bakamjian-Thomas construction [1]. Its point-form version amounts to the assumption that the (interacting) 4-momentum operator $\hat{P}^\mu$ can be factorized into an interacting mass operator and a free 4-velocity operator

$$\hat{P}^\mu = \hat{M}\hat{V}^\mu_{\text{free}} \,. \tag{1}$$

It is therefore only necessary to study an eigenvalue problem for the mass operator.

In case of elastic electron-meson scattering a mass eigenstate $\hat{M}|\psi\rangle = m|\psi\rangle$ is written as a direct sum of a quark-antiquark-electron component $|\psi_{Q\bar{q}e}\rangle$ and a quark-antiquark-electron-photon component $|\psi_{Q\bar{q}e\gamma}\rangle$. Here we have already assumed that the quark carries the heavy flavor. The mass eigenvalue equation to be solved has the form

$$\begin{pmatrix} \hat{M}_{Q\bar{q}e} & \hat{K} \\ \hat{K}^\dagger & \hat{M}_{Q\bar{q}e\gamma} \end{pmatrix} \begin{pmatrix} |\psi_{Q\bar{q}e}\rangle \\ |\psi_{Q\bar{q}e\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{Q\bar{q}e}\rangle \\ |\psi_{Q\bar{q}e\gamma}\rangle \end{pmatrix} , \tag{2}$$

where $M_{Q\bar{q}e}$ and $M_{Q\bar{q}e\gamma}$ consist of a kinetic term and an instantaneous confining potential between quark and antiquark, and $\hat{K}$ is a vertex operator which accounts for the emission and absorption of a photon by the electron or (anti)quark. It is determined by the interaction Lagrangean density of QED [2].

For the calculation of the electromagnetic meson currents and form factors it is most convenient to apply a Feshbach reduction to the mass eigenvalue problem

$$(\hat{M}_{Q\bar{q}e} - m)|\psi_{Q\bar{q}e}\rangle = \underbrace{\hat{K}^\dagger (\hat{M}_{Q\bar{q}e\gamma} - m)^{-1} \hat{K}}_{\hat{V}_{\mathrm{opt}}(m)} |\psi_{Q\bar{q}e}\rangle \tag{3}$$

and study the optical potential $\hat{V}_{\mathrm{opt}}(m)$. The electromagnetic meson current $J^\mu(\vec{k}'_M;\vec{k}_M)$ can then be extracted from the invariant 1-γ-exchange amplitude which is essentially given by on-shell matrix elements of the optical potential. These have the structure

$$\begin{aligned}\mathcal{M}_{1\gamma}(\vec{k}'_e,\mu'_e;\vec{k}_e,\mu_e) &\propto \langle V';\vec{k}'_e,\mu'_e;\vec{k}'_M|\hat{V}_{\mathrm{opt}}(m)|V;\vec{k}_e,\mu_e;\vec{k}_M\rangle_{\mathrm{on-shell}} \\ &\propto V^0\delta^3(\vec{V}-\vec{V}')\frac{j_\mu(\vec{k}'_e,\mu'_e;\vec{k}_e,\mu_e)J^\mu(\vec{k}'_M;\vec{k}_M)}{(k'_e-k_e)^2}\,. \end{aligned} \tag{4}$$

$|V;\vec{k}_e^{(\prime)},\mu_e^{(\prime)};\vec{k}_M^{(\prime)}\rangle$ are, so called, "velocity states" that specify the state of a system by the overall velocity and the center-of-mass momenta and canonical spins of its components [3]. In our case $\vec{k}_M^{(\prime)}$ is the momentum of the confined q-$\bar{q}$ subsystem with the quantum numbers of the heavy-light meson. "On-shell" means that $m = k_e^0 + k_M^0 = k_e'^0 + k_M'^0$ and $k_e^0 = k_e'^0$, $k_M^0 = k_M'^0$. A detailed derivation of Eq. (4) and the explicit expression for the meson current $J^\mu(\vec{k}'_M;\vec{k}_M)$ can be found in Ref. [4].

If we are dealing with a pseudoscalar meson its electromagnetic current should be of the form $J^\mu(\vec{k}'_M;\vec{k}_M) = (k'_M + k_M)^\mu F(Q^2)$, which allows us to identify the electromagnetic form factor of the meson uniquely. It is, however, known that the Bakamjian-Thomas construction, that we are using, provides wrong cluster properties [5]. As a consequence, the hadronic current $J^\mu(\vec{k}'_M;\vec{k}_M)$ which we extract from Eq. (4) exhibits a slight dependence on the electron momenta k_e and k'_e [1]. Fortunately this dependence vanishes rather quickly with increasing invariant mass m of the electron-meson system and thus also in the heavy-quark limit ($m_Q = m_M \to \infty$, $m_{\bar{q}}/m_Q \to 0$). This limit has to be taken in such a way that $v' \cdot v = 1 + Q^2/2m_M^2$ stays constant. The function of $(v' \cdot v)$ that is obtained from $F(Q^2)$ by taking the heavy-quark limit is the famous Isgur-Wise function [7]. In our case it takes on a rather simple analytical form:

$$\begin{aligned}&\xi_{EM}(v'\cdot v) = \lim_{m_Q\to\infty} F(Q^2) \\ &= \sum_{\mu'\mu}\int d^3\tilde{k}'_{\bar{q}}\sqrt{\frac{\tilde{\omega}_{\bar{q}}}{\tilde{\omega}'_{\bar{q}}}}\sqrt{\frac{2}{1+v\cdot v'}}\frac{1}{2}D^{1/2}_{\mu'\mu}\left[R_W^{-1}\left(\frac{\tilde{k}_{\bar{q}}}{m_{\bar{q}}},B(v_{Q\bar{q}})\right)R_W\left(\frac{\tilde{k}'_{\bar{q}}}{m_{\bar{q}}},B(v'_{Q\bar{q}})\right)\right] \\ &\quad\times\psi_{\mathrm{out}}(\vec{\tilde{k}}'_{\bar{q}})\,\psi_{\mathrm{in}}(\vec{\tilde{k}}_{\bar{q}})\,. \end{aligned} \tag{5}$$

[1] See Ref. [6] for a short discussion of this problem.

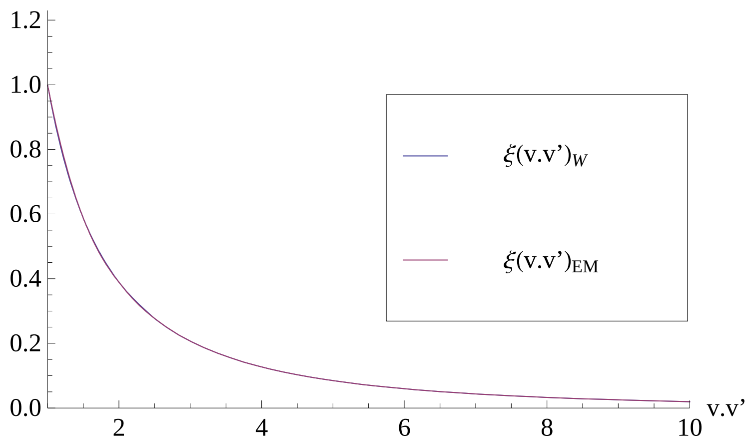

Figure 1: Isgur-Wise function for a heavy-light meson as obtained from electron-meson scattering (cf. Eq. (5)) and semileptonic decays (cf. Eq. (6)). For the Q-$\bar{q}$ bound-state wave function we have taken a Gaussian with the same oscillator parameter ($a = 0.55$ GeV) and light-quark mass ($m_{u,d} = 0.25$ GeV) as in Ref. [9].

It is just an integral over incoming and outgoing wave functions, a Wigner-rotation factor and kinematical factors. The tildes in the integral indicate that the corresponding quantities are given in the Q-$\bar{q}$ rest system. In accordance with heavy-quark symmetry $\xi_{EM}(v' \cdot v)$ does not depend on the heavy-quark mass.
Heavy-quark symmetry allows us also to relate the electromagnetic form factor of a pseudoscalar heavy-light meson to its weak decay form factors for heavy-to-heavy flavor transitions (like, e.g., $\bar{B} \to D^{(*)}\ell\bar{\nu}$). In the heavy-quark limit the electromagnetic form factor and the weak decay form factors (modulo kinematical factors) should lead to only one Isgur-Wise function [8]. If we apply our coupled channel framework to semileptonic decays of pseudoscalar heavy-light mesons, identify the decay form factors from the optical potential and take the heavy-quark limit we end up with

$$\begin{aligned}\xi_W(v' \cdot v) \quad &= \quad \sum_{\mu'\mu} \int d^3\tilde{k}'_{\bar{u}} \sqrt{\frac{\tilde{\omega}_{\bar{u}}}{\tilde{\omega}'_{\bar{u}}}} \sqrt{\frac{2}{1+v'\cdot v}} \frac{1}{2} D^{1/2}_{\mu'\mu} \left[R_W \left(\frac{\vec{\tilde{k}}'_{\bar{u}}}{m_{\bar{u}}}, B(v'_{c\bar{u}}) \right) \right] \\ &\quad \times \psi_{\mathrm{out}}(\vec{\tilde{k}}'_{\bar{u}}) \psi_{\mathrm{in}}(\vec{\tilde{k}}_{\bar{u}}) \,. \end{aligned} \tag{6}$$

At first sight $\xi_{EM}(v' \cdot v)$ and $\xi_W(v' \cdot v)$ seem to be different and we are still not able to show their equality analytically. A numerical study, however, reveals that they coincide (see Fig.1). These investigations show that heavy-quark symmetry is recovered in the heavy-quark limit within our relativistic coupled channel approach.
It is also interesting to study the breaking of heavy-quark symmetry caused by finite values of the heavy-quark mass. This is done in Fig. 2 for the two weak decay

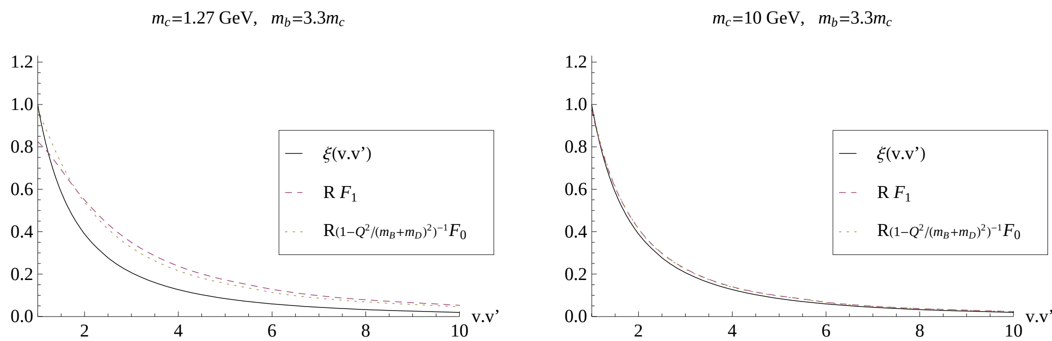

Figure 2: Weak decay form factors (multiplied with appropriate kinematical factors) for the process $B^- \to D^0 e^- \bar{\nu}$ for finite m_Q in comparison with the Isgur-Wise function. The wave-function parameterization is the same as in Fig. 1.

form factors $F_0(v' \cdot v)$ and $F_1(v' \cdot v)$ that show up in the semileptonic $B^- \to D^0 e^- \bar{\nu}$ decay. If heavy-quark symmetry was perfect RF_1 and $R(1-q^2/(m_B+m_D)^2)^{-1}F_0$ (with $R = 2\sqrt{m_B m_D}/(m_B + m_D)$) should coincide with the Isgur-Wise function $\xi(v' \cdot v)$ (see Ref.[8]). What we rather observe is that the physical values of the b- and c-quark mass give rise to a considerable breaking of heavy-quark symmetry (left plot). Here we have not even taken into account a (heavy) flavor dependence of the meson wave functions. If both masses were about a factor of 10 larger heavy-quark symmetry would nearly hold (right plot).
A more comprehensive study of heavy-light systems along the lines presented here, including the discussion of (heavy-quark) spin symmetry, can be found in Ref. [4].

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